Name:_____ November 1, 2013

Geometry: Building Triangles in SPACE!

1 Gravitational Waves

Gravitational waves are ripples in space produced when something very massive starts moving very fast. When two **black holes** orbit around one another, they produce gravitational waves.

2 LISA: The Laser Interferometer Space Antenea

The *Laser Interferometer Space Antenea* (or **LISA**) is a proposed satellite system to measure gravitational waves. When undisturbed, the three satellites make an equilateral triangle with sides that are 5 million kilometers long.

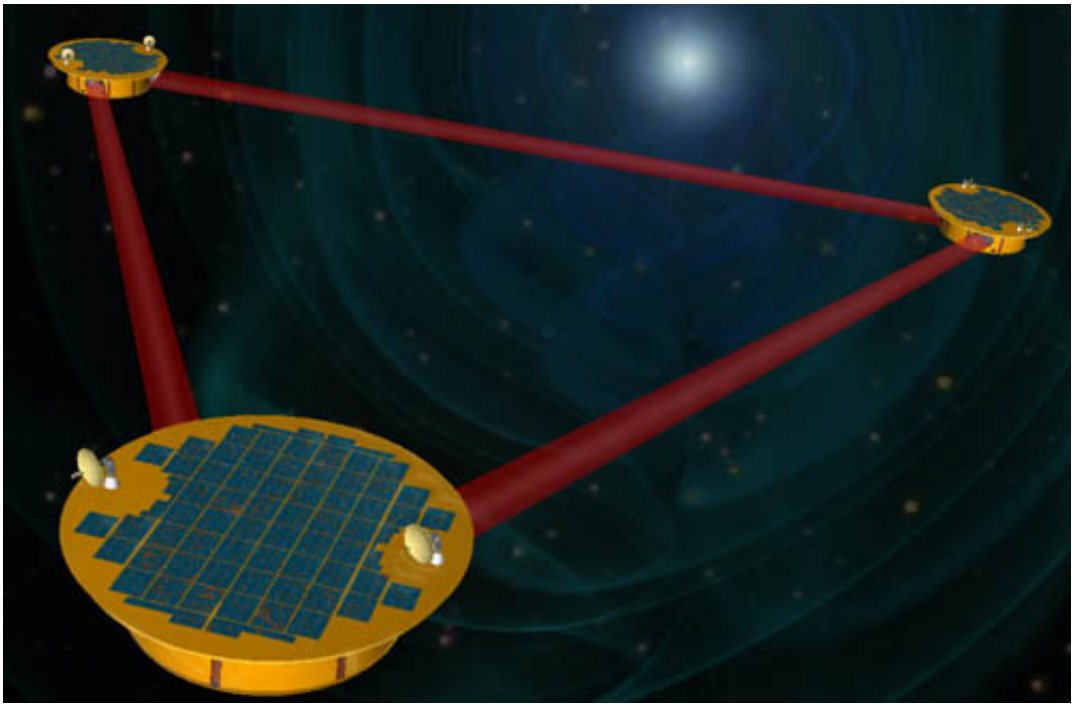


Figure 1: The LISA Constellation in Orbit (courtesy of lisa.nasa.gov)

When a gravitational-wave from two black holes passes through LISA, it causes the triangle to change shape:

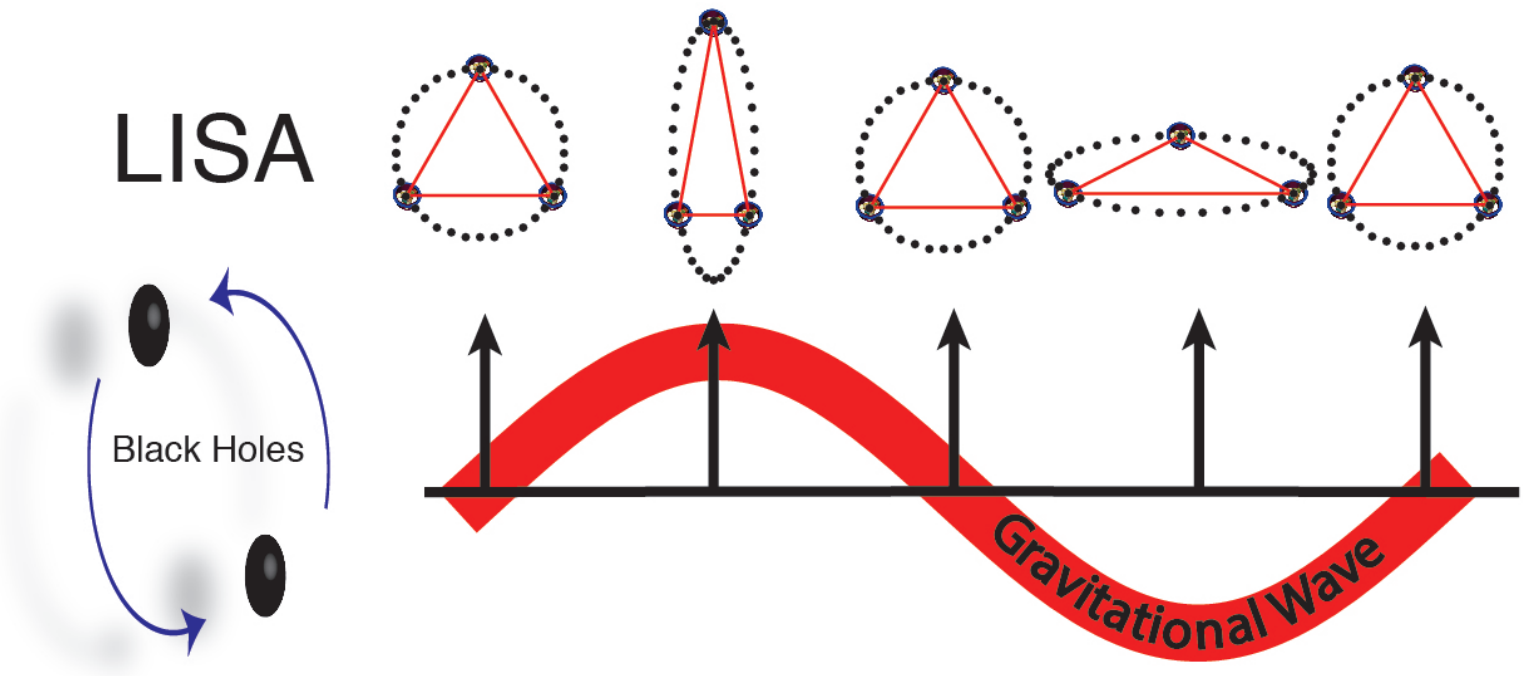
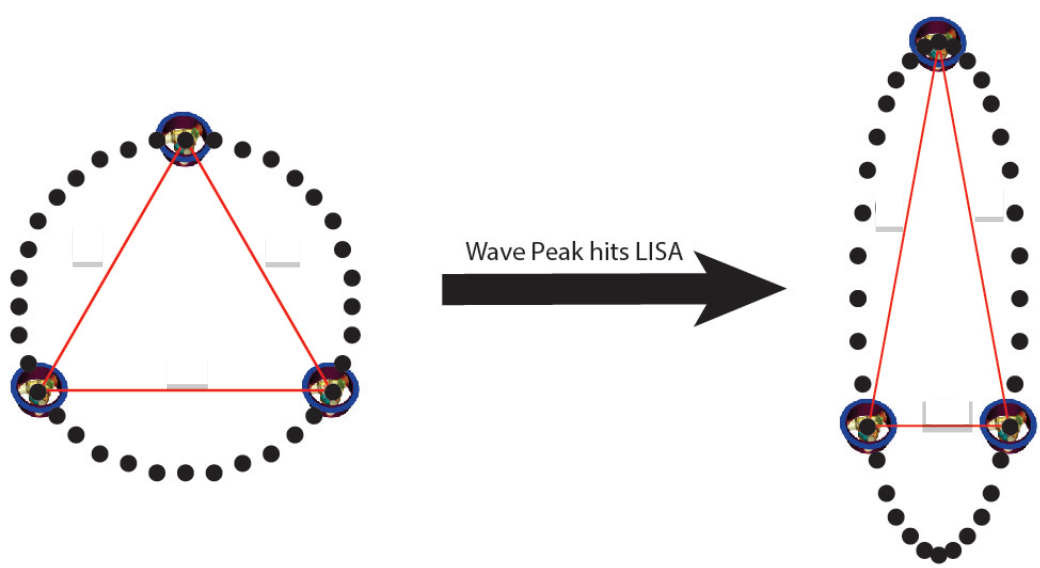


Figure 2: What happens to the LISA Constellation when a gravitational wave passes through it.

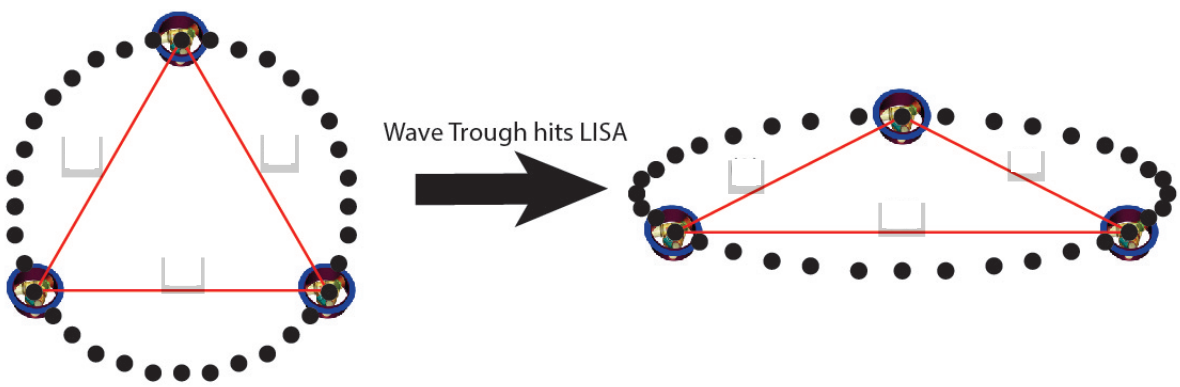
1. When a gravitational-wave passes through the LISA Constellation, it shrinks one side of the triangle and stretches the other two:



If sides (A) and (B) get longer, and side (C) shrinks, what kind of triangle does the LISA Constellation form?

2. If side (C) shrinks to 4 million kilometers and sides (A) and (B) grow to 6 million kilometers, what do the following angles become:
   - $\angle(A)(C)$?
   - $\angle(A)(B)$?

3. When the wave trough hits LISA, the triangle becomes:



If sides (A) and (B) get slightly smaller, but side (C) gets much larger, what kind of triangle does the constellation become?

4. If side (C) grows to 7 million kilometers and sides (A) and (B) shrink to 4.5 million kilometers, what do the following angles become:
   - $\angle(A)(C)$?
   - $\angle(A)(B)$?

3 Measuring Black Holes with Triangles

Most spiral galaxies contain a **supermassive black hole** in the center. Supermassive black holes are millions to billions of times as big as our sun. When two spiral galaxies collide, their black holes can start orbiting one another:

Figure 3: Spiral Galaxies NGC 5426 and NGC 5427 colliding in deep space. Image from the Gemini-South Telescope in Chile (http://apod.nasa.gov/apod/ap130825.html)

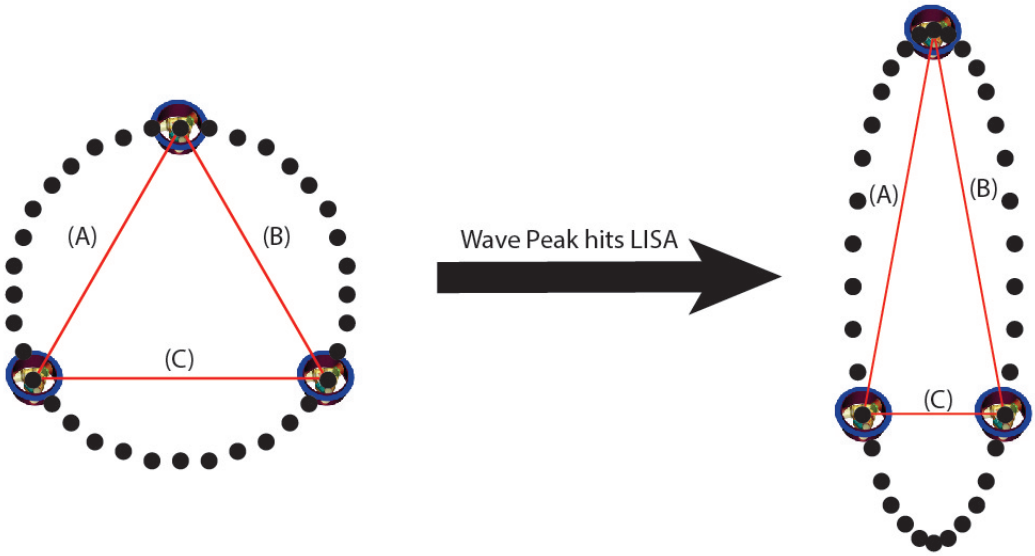
By measuring how much the gravitational waves change the LISA telescope, I can work out how massive the black holes are.

Let's say the black holes are about 10 billion meters apart and a thousand light years from Earth. Then the **mass squared** in the arm of the detectors will be

$$M^2 = \frac{D(C)}{(C)} \cdot 4.336 \times 10^{22} M_\odot^2 \tag{1}$$

We're going to use this to calculate the masses of black holes using LISA.

Lets say I measure LISA when it's an isosceles triangle



and now the angle $(A)\angle(B) = 59.9977^o$ and the lengths of (A) and (B) are 5,000,060km.

1. What is (C), in kilometers?

2. What is D(C), the difference in length of side (C) from what it was before the gravitational wave hit?

3. What is the **fractional difference**? Just divide your answer from question 2 by your answer from question 1.

$$\frac{D(C)}{(C)} =$$

4. Using equation (1), what are the mass of the two black holes?