Name:______________________________ May 8, 2014

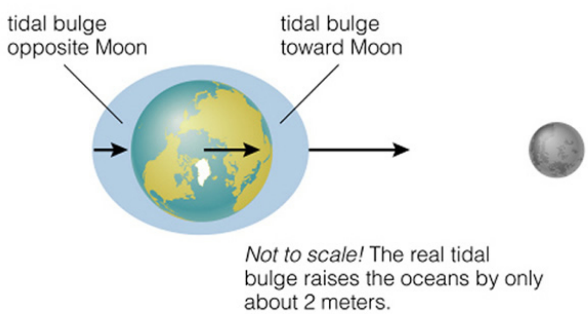
Algebra II/Trig: Tides

1 Tides on Earth

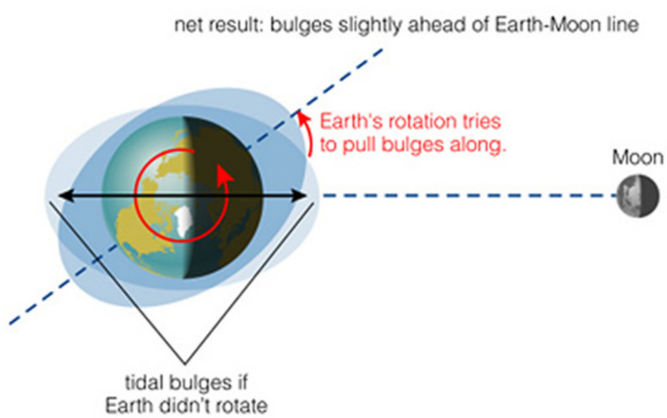
Tides on Earth are caused by the gravitational pull of the moon. Gravity is inversely proportional to the square of the distance between two objects:

$$F = G\frac{mM}{D^2}$$

Since the distance between the moon is not the same on either side of Earth, the force of gravity is greater on the side facing the moon. This difference in gravitational force is what causes tides:



Since the Earth rotates faster on its axis (once per day) than the moon orbits around the Earth (once every 27.3 days), the tides on Earth actually get rotated forward, causing the water to rush ahead of the moon:



Because the two bulges now pull on the moon at different angles, the tides on Earth torque the planet, causing the Earth's rotation to slow down.

2 Modeling Tides

When we talk about tides, we use a simplified model, where we treat each of the tidal bulges as a small mass:

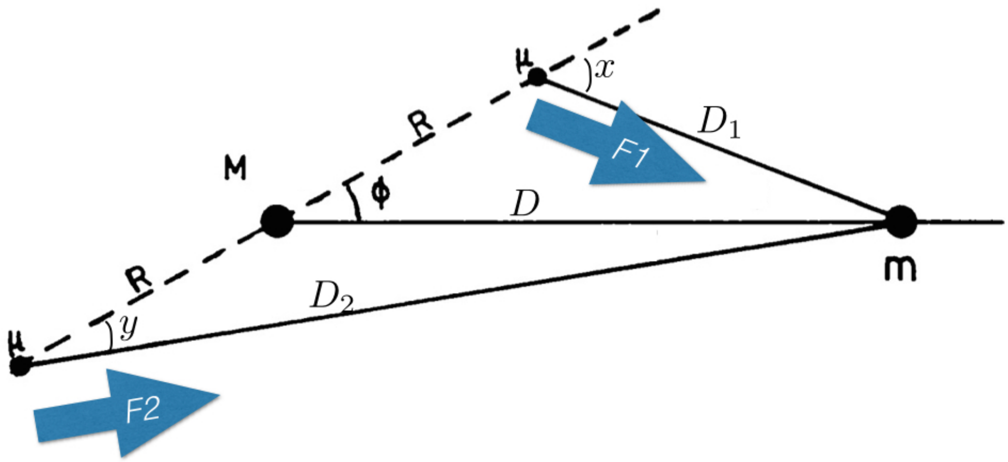


Figure 1: Model for how tides cause forces on Earth (Hut 1981)

We want to know what force the tides exert on the moon. We know that the moon is (on average) 3.844×10^5 km from Earth. We also know from Oceanographers that the surface of the oceans is about 6378.1 km above the center of the Earth, and that the rotation of the Earth pulls the tidal bulge in front of the Earth by about 0.4 degrees.

Question 1

What we need to know is how far each tidal bulge is from the moon, in order to find out what the force of gravity is beween them.

Using whatever technique you want (law of sines, law of cosines, etc), find the distances D_1 and D_2 and the angles x and y

Question 2

We know that the Earth has a mass of 6×10^{24} kg, and the moon has a mass of 7.3×10^{22} kg. If $G = 6.67384 \times 10^{-11} \frac{m^3}{kg \times s^2}$, and the mass of the tides is $\mu = 7.86 \times 10^{12}$ kg, what are $F1$ and $F2$?

Question 3

Finally, use your results, and the following equation, to find α, the rate at which the rotation of Earth is slowing.

$$F1 \sin(x) - F2 \sin(y) = \frac{2}{5} M R \alpha$$

Your answer will be in units of $\frac{1}{s^2}$. Convert it to units of s to find out how much of a day we loose to tides every century. To do that, multiply by

$$\alpha \cdot 4.3 \times 10^{23} s^3 =$$