

## Algebra II: More Flight Paths over North America

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### 1 Airports in North America

Take a look at the following, ridiculously simplified, map of airports in the United States:

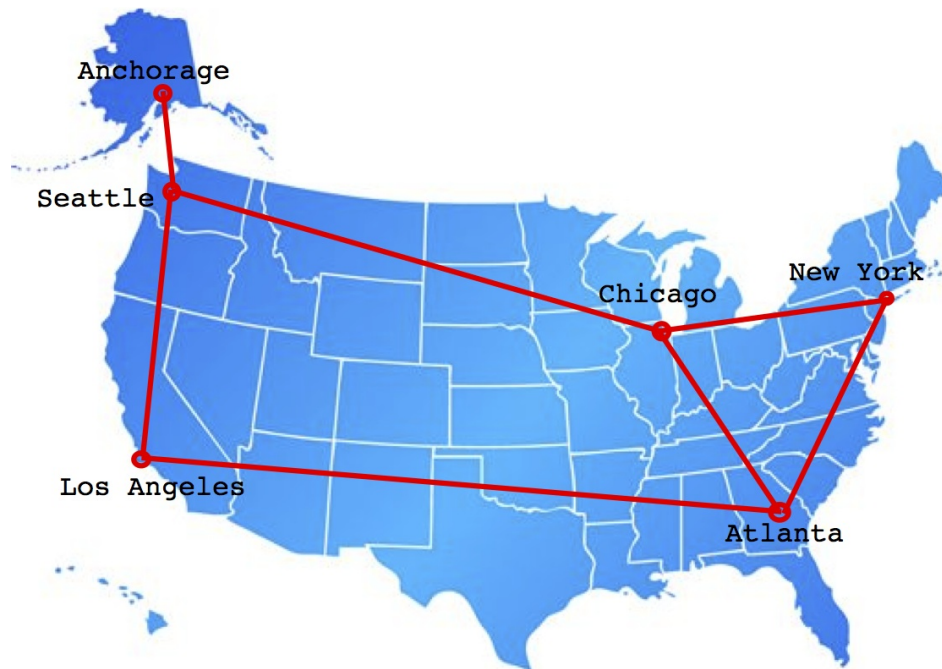


Figure 1: A substantially reduced version of airport flights in the US

- Write down the **adjacency matrix** for the 6 airports and the routes between them. For each column/row in the matrix, put a 1 if a flight goes from the city in the row to the city in the column. Put a 0 if there is no flight.

$$\underline{\mathbf{A}} = \begin{array}{c} \textit{Anc} \\ \textit{Sea} \\ \textit{LA} \\ \textit{Chi} \\ \textit{NY} \\ \textit{Atl} \end{array} \begin{bmatrix} \textit{Anc} & \textit{Sea} & \textit{LA} & \textit{Chi} & \textit{NY} & \textit{Atl} \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix}$$

2. Now square the adjacency matrix using your calculator

$$\underline{\mathbf{A}}^2 = \underline{\mathbf{A}} \times \underline{\mathbf{A}} = \begin{matrix} & \begin{matrix} Anc & Sea & LA & Chi & NY & Atl \end{matrix} \\ \begin{matrix} Anc \\ Sea \\ LA \\ Chi \\ NY \\ Atl \end{matrix} & \left[ \begin{array}{cccccc} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{array} \right] \end{matrix}$$

3. Which cities can I not get between in two jumps?

4. How many flights would I need before I could get between every city.

(a) What two cities would need the most flights to fly between?

(b) Compute higher powers of  $\underline{\mathbf{A}}$  ( $\underline{\mathbf{A}}^2$ ,  $\underline{\mathbf{A}}^3$ , *etc.*) using your calculator, until the number for those two cities is not 0. How many flights do I need before I can get between those two cities?